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From a Dynamical to a Semiotic Account of Emergence

Comments on Collier's Paper "A Dynamical Account of Emergence"

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Collier offers a physicalist account of emergence that I welcome. Emergence is, thus explained as a property of non Hamiltonian, non Holonomic physical systems and can be detected by its non computability. Emergence constitutes a necessary and sufficient condition for the emergence of living systems or agents that process information and generate the corresponding informative records. By distinguishing between analog and digital information processing, I will show that analog information processing corresponds to a physical property of radical non Hamiltonian, non Holonomic systems that are responsible for emergence in general terms, but also more specifically to living systems that process digital information. In the way from dynamics to semiosis an intermediary model of physical systems that process both analog and digital information is needed. In consequence a contrast between Collier's Physical Information Systems (PIS) and Zurek's Information Gathering and Using Systems (IGUS) will be advanced in order to show that the latter best accounts for the origin of semiosis for it implicitly assumes that information from the environment can be recorded by means of structural adjustments that create the conditions for the appearance and selection of digital informative records. These systems fulfill the properties of a Peircean sign because they are dynamical systems that serve as a medium for the transmission of form. Thence individuation, autonomy and teleology are best understood as the outcome of information processing, made possible by dynamics.

Comments on Collier's dynamical account of emergence

Against the current prejudice according to which, irreducibility to atomic components is beyond physical account, Collier proposes the understanding of emergence in physical terms by showing that it also applies to physical systems. In this quest Collier follows the path followed by a number of authors like Volkestein (1994), Kauffman (1993), Salthe (1993), and Rosen (2000) just to name a few. My aim will be to examine the applicability of Collier's tenets to the study the emergence of living systems, in particular the origin of life one of the most puzzling emergent phenomena ever occurred. Despite the fact that explanations of emergence in terms of substance

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are no longer relevant, vitalistic approaches have not been completely dispelled since the appearance of new properties is often conceived as a sudden jump, providing the false impression that the continuity of nature was severed at least on very rare occasions. I hail Collier in taking the challenge of offering a physical account of emergence because it highlights the principle of continuity. The preservation of this principle that Peirce named as *synechism* or "the tendency to regard everything as continuous" (C.P. 7.565) is the best way to overcome the traditional polarity between reduction and emergence.

In previous papers, Collier asks about the ontological status of the different levels that are organized in a pattern of nested hierarchies (Collier & Muller, 1998), now he proposes a dynamical account of emergence. He starts by examining the conditions for a physical ontological reductionism: R1. The existence of "logical constructs" that are made to correspond to real atomic entities. R2. Closure or independence from external fluctuations. R3. Finitism, or finite computation. Therefore, reductionism and predictability require artificial logical constructions like the assumption of atomic entities (R1) and artificial closure given by the definition of boundary conditions or fixed environmental parameters (R2). If these two conditions are met, it follows that there must be computability (R3). As stated by Collier these are not conditions of a Newtonian systems but the result of the Lagrangian formulation that assumes the complete transformation of potential into kinetic energy in agreement with the principle of energy conservation, ruling out losses by friction and energy exchanges with dissipation. Reductionism according to Collier is a theoretical possibility only for closed Hamiltonian, holonomic systems, acted by conservative forces so that their description depends solely on the position coordinates and time. Consequently a dynamical linear superposition of trajectories that are not mutually affected, would allow an analytic decomposition and reduction applicable even for ideal irreversible processes with temporal asymmetry, as in Boltzmann approach.

It is important to be reminded of Peirce's views about the principle of energy conservation that lies behind reductionist approaches.

> Still, as a scientific generalization, it [the law of the conservation of energy] can only be a probable approximate statement, open to future possible correction. In its application to the ordinary transformation of forces, it has been pretty exactly verified. But as to what takes place within organized bodies, the positive evidence is unsatisfactory, and, in connection with the question of free will, we cannot feel sure the principle holds good without assuming a partisan position which would be unwise and unscientific. (Peirce, N2, 115)

> The law of conservation of energy is equivalent to the proposition that all operations governed by mechanical laws are reversible; so that an immediate corollary from it is that growth is not explicable by those laws, even if they be not violated in the process of growth. (Peirce, C.P. 6.14)

If the principle of energy conservation does not apply for organized bodies, reductionist approaches will not do either. Computation requires ideal conditions given by the assumption that forces act conservatively, thus neglecting the effect of friction and dissipation. If a conservative system requires very large computability like in the case of the "three body problem", still it is computable. Predictability is a direct outcome of condition R1 and R2, however if we lift one of the above conditions, reduction becomes impossible. Moreover reductionism cannot account for the coupling between genotype (lower level informational possibilities), and phenotype (organizing higher level entities) in the context of a specific environment. Nonetheless the claim that life is an irreducible phenomenon does not imply that it cannot be physically explained, all the contrary, it only points out at the insufficiency of mechanical approximations.

By negating one by one the above conditions of reduction, Collier (2008) gets the following conditions of emergence: E1. Non-reducibility to components. E2. Openness to external fluctuations. Likewise, if E1 and E2 are met, thence it follows (E3) non computability and unpredictability. Therefore, one must tackle the problem by focusing on the computational conditions of emergence that is "to determine when computation fails". As he says, "the understanding of emergence becomes the problem of how to hook dynamical conditions to computation" (Collier, 2008). In this point Collier follows the road paved by Rosen (2000) when he stated that the most distinct characteristic of a complex system is its non-computability since the causal entailments on which they depend cannot be encoded into the models we are using to compute it.

In his definition of "dynamical realism", Collier (2008) is equating emergent systems with complex systems, where everything real is dynamic and therefore can induce an effect on anything else. Such a dynamical world would manifest [illegible] complex network of interconnected entities acting on each other, making meaning possible. Therefore, emergent phenomena result from the non-linear interactions of their components that induce new system dynamics.

I will put the emergence problem in the following way: let us have a physical system A that acts as a lower level and let B be the higher level. "A brings forth B", but "B cannot be explained solely in terms of A", or "B is not deducible from A". The reduction of B in terms of A becomes impossible because something new in B has emerged. Where do novelties come from? What is new in B and was absent in A is its *form* or organization pattern that results from the interaction of A with its surroundings (E). Then, A + E = B. But, E is currently obliterated in classical approaches that assume the stability and constancy of fixed boundary conditions (environmental parameters). Thence, artificial closure or the isolation from the environment is basically the source of the emergence problem. The emergence problem appears every time that one tries to understand the passage from A to B disregarding E. This fact explains why there cannot be emergence in a closed system (Andrade 2003). The search for causal explanations of emergence involve the environment, since the interactions among cohesive parts always occurs in the context of a specific environment. Open ended evolutionary physical systems cannot be accounted solely in terms of their basic constitutive components. Consequently the understanding of emergence would permit to deal with the problem of how a hierarchical system is organized, more specifically how genotype phenotype coupling is achieved, for instance between protein sequence and protein structure (Balbin & Andrade 2004).

Collier (2008) in his search for the limits of computation argues that the conditions of radically, (I would prefer far from equilibrium) non-Hamiltonian behavior that underlie all emergences are: 1. the presence of non holonomic constraints. 2. Energetic and informational openness. 3. The existence of multiple attractors. 4. The characteristic rate of at least one property of the system is of the same order as the rate of the non holonomic constraint. 5. A system is emergent if at least one of the properties is an essential property of the system, in other words if at least one of the properties is irreducible.

With reference to condition 1, non computable, non integrable system must be non holonomic, in other words its computation requires an additional non holonomic constraint expressed as a rate of change dependent on the path it actually follows that is usually a function of environmental parameters, so that the number of coordinates required to represent them completely is more than their degrees of freedom. With regard to condition 2, openness allows the emerging system's capability to probe (measure and interact) the environment. In this manner information processing systems gather information about energy resources that are to be captured and spent in structural organization and maintenance, as well as in keeping the capacity to further probing the environment in search of newly available energy resources. With regard to 3, it is worth to note that whereas for closed systems, multiple microstates correspond one equilibrium macrostate, for open systems, to one specific set of microstates may correspond different macrostates depending on environmental parameters. Therefore, I do not think that the third condition is debatable, even if it seems not to apply to Benard's cells, on the contrary it is an essential condition that implies that for a given lower level informational source, and there can be more than one attractor that characterizes the higher level.

Complex emergent systems are unpredictable not only epistemologically, but also ontologically. According to the former, the slightest uncertainty in the knowledge of the state of a system in a present time leads to an informational lost about its future states, according to the latter there are events highly sensible to initial conditions that make impossible the exact determination of these conditions and the predictability of future states, (Barrow, 1991, p. 167-168). This latter case applies best to far from equilibrium, non Hamiltonian systems that cannot be computed and are emergent though they are entirely physical. A rolling wheel submitted to friction and a planet that experiences tidal dissipation like the sun-mercury system, exhibit emerging properties that result from rapidly changing parameters. However these cases might not be easily extrapolated to explain the passage from non-living to living systems, since something else not included in dynamics but causally dependent on it, must be included. I will develop this idea below.

Collier (2008) makes also use of Humphrey's approach according to which properties are emergent if they cannot be computed from the properties of the unfused components. A fused system that can be computed from the properties of the unfused components is not emergent. Cohesive objects are dynamical systems. Novelty emerges as a result of fusion of some preexisting components. The emergent entity is a system. Methodologically it is wise to assume that most phenomena are mere manifestation of the underlying level (epiphenomena) until the basis of their cohesion or causal interactions among the parts is fully explained. Cohesion, thus, represents the set of factors that causally link the components through space and time, resulting in coherent behavior and resilience to external and inner fluctuations. When Collier defends that "water, if it is emergent, is emergent because of the facts about water, no about our theories of water" (Collier & Muller, 1998 [need page number for quotation]), he is assuming a degree of internal cohesion that is responsible for causal interactions among the components of water. For him water description requires parameters not needed to describe H and O separately.

Collier correctly concludes that emergence must be defined in terms of causal relations that permit to uncover causal autonomy, novelty, irreducibility and unpredictability of emergent phenomena. Cohesion is the base of dynamical identity that is a key distinctive feature of emergent systems, for they are more cohesive with themselves than with any other subsystem, since they are more stable and resistant to random internal and external noise. Cohesion is dependent on the system's physical context, and is not a subjective matter.

> Cohesive objects have properties that are not merely effects of the properties of their components (though they are too). They differ from epiphenomenal objects in that, far from being merely effects, they can act as causes as well, independent of the particular contingencies of their composite parts. (Collier, 2008, page number?)

Can higher levels be logically reduced to lower ones? From a logical stand point the description of new properties requires new parameters not needed to describe the lower level. Therefore candidates for hierarchical emergence are molecules, dissipative structures, chaotic systems, self-organizing systems, autocatalytic phenomena, biological functions, human communities, etc., but agreeing with Collier (2008), for each level it is, thus, crucial to demonstrate that they are not just logically inferred but a real cohesive dynamical systems. Also, I agree with Collier (2003) that hierarchical levels are not just mere aggregates (scalar hierarchies) or logical categories (specific hierarchies) in the sense of Salthe (1993), but real cohesive entities. Nonetheless Collier (2003) is wrong to criticize Salthe for not considering hierarchies as cohesive systems. In Salthe (1993, p. 152) it is clearly stated that the development of a self organizing system can be explained in terms of changing connectivity patterns going from immature (over-connected with weak communications between almost all the components) to mature (less connected with more definite hubs that make a dominant loop with stronger cohesive links) to end up in senescence (appearance of redundant loops with diminishing strength). Furthermore, Salthe explains how self-organizing systems support the emergence of a hierarchical organization by the action of internal forces. More explicitly emergence is referred to as a sudden increase (or decrease) in system's cohesion (Salthe, 1993, p. 206).

> When internal forces are strong, as in immature systems, they determine the results; however, as these forces weakens, as a system moves toward equilibrium, idiosyncrasies can come into play that were suppressed by the central tendency of the far from equilibrium situation. (Salthe, 1993, p. 202).

[Why are these page numbers different? In this case, the quotation appears to be an extension of what it follows]

To summarize, the characteristics of emergent phenomena are: E1) Irreducibility to the properties of the lower level. Reductionism invokes the information necessary to specify the identity and macroscopic properties of an object, by identifying what is reduced (higher level) to the reductive components (lower level). Cohesion explains why it is not possible to do that. E2) Individuality, so that the whole becomes a unity as a result of internal binding forces expressed as causal interactions that make it resistant to external and internal fluctuations. Cohesion accounts for individuality or the space temporal continuity of an entity with high specificity and stability that exerts an impact on the environment. E3) Downward causation in the sense that the dynamics of the whole constraints lower components dynamics. E4) Unpredictability and novelty. E5) In order to account for living systems it is needed to add information processing with the ensuing production of informative records of a digital character like DNA, RNA, and proteins.

The emergence of living systems

Collier asserts that individuality is a self-sustained form of cohesion, with its components contributing to the maintenance of the autonomy. If the maintenance of closure is assured autonomy survives. In this vein he states rightly that cohesion accounts for autonomy as an emergent characteristic and with it functionality and teleology can be understood as the result of comprehensive dynamics. *Autopoiesis* commonly considered as an exclusive property of living entities is associated to autonomy and requires an organizational closure. According to Collier dynamics opens the way to semiosis; this seems to be a bold leap that yet has to be bridged with an intermediary model of an information processing agency, in order to avoid the current misinterpretations that consider semiosis as the last trench of vitalism. In the road to semiosis the ideas of Collier's (2003) PIS, Zurek's (1989, 1990) IGUS, and Andrade's (2007) Evolving Developing Agents (EDA), whatever their limitations aim to conceptualize physical systems that are semiotic. Semiosis is here understood as the theory of signs. It suffices here to remind that a *sign* is a dynamical system that serves as a medium for the transmission of a form (information) (Queiroz & El-Hani, 2006). The mediation between an informative record (DNA) and the corresponding form (phenotype) is made possible by the action of an interpretative agent that played the roll all along development and evolution of a code maker (Barbieri, 2003), (Andrade's 2007). In a more general sense semiotic agents (interpretants ~~in the sense of~~ sensu? Peirce) not only encode information, but record, replicate, transfer, translate, interpret, use, preserve, alter, erase, and dispose it.

Collier is right in insisting that emergence is a physical causal process. But considering that living systems collect, record and use environmental information in order to extract free energy, the question turns out to be whether information processing can be considered a form of causation. In order to answer this question one must be clear about what is meant by information processing. If we accept Hoffmeyer and Emmeche's (1991) distinction between analog and digital information, then the term *information processing* has two aspects.

A) Analog information processing. Living beings are communication systems that are integrated as a whole within its own boundaries by signals, but also they are integrated to their environment by interpreting and reacting to external signals (i.e. the state of a cell is modified by the presence of a determinate signal). Analog processing of information refers to what Root-Berstein and Dillon (1997) define as complementarity that embraces all kind of cohesive forces that permits the formation of non random stable aggregates, that is at the molecular level complementarity is best understood as chemical affinity.

The point I want to address is that analog information thus described is the source of cohesion, becoming the crucial step in emergence (Andrade, 2003). This form of information processing by non-random interactions between constitutive components, is the causal agency of the formation of macromolecular aggregates that generate organelles, prokaryotic cells that interact to generate eukaryotic cells, cells that interact to generate tissues, and so forth, so leading to the emergence of a new ontological level with a distinctive qualitative behavior that is characteristic of a new organized functional structure that acts as a coherent whole. Consequently analog driven emergence (Andrade, 2003) is the result of the establishment of new cohesive analog-analog interactions between already existing components constrained by environmental conditions. This type of emergence provides the clue to approach the study of evolutionary transitions like the origin of life (Root-Berstein & Dyllon, 1997; Fox, 1984). In agreement with Collier, cohesion results in closure of causal relations among the dynamical parts of a dynamical structure that determines its resistance to external and internal fluctuations. Semiosis accepts the existence of physical agents for whom information is meaningful. Of course that agents may act randomly, but the point is that as information is gathered and encoded their behavior becomes more directed to specific goals. The introduction of self referential agents for whom information is meaningful creates a closed loop that makes computation impossible. This way of interpreting information cannot be formalized in purely syntactic terms because it expresses the self-referential generative process of an agent with its partially describable external environment. Agents' capacities to choose among a random set of components those with which they will interact are the very source of creative and unpredictable interactions with external referents. Emergence exists because new unforeseen semiotic relations can always be established.

B) Digital information processing. Digital information corresponds to what after Shannon and Crick is properly understood as information and suits very well Collier's (2003) concept of lower level information. Digital information refers to encoded information in the form of a record that has the structure of a text composed of basic symbols (DNA) that can be modified by discontinuous variations such as mutations and recombination. But the processing of this information refers to a more abstract operation in which a polymer built out of monomers that are equivalent to symbols, are formed and selected not only for their catalytic functions (chemical affinity) but also for their use as replicating templates that streamline the production of more polymers. This is the starting point of a genetic record that characterizes life as we know it. Once specific sequences are selected for the functionality of their encoded products to the extent that they contribute to cell maintenance and reproduction, the coupling phenotype genotype is produced. Digital encoding of information facilitates the emergence of new functions by permutations of existing strings, only if the new information is expressed in the phenotype and functionally selected.

These two ways of processing information (analog or selection by structural complementarities and digital through the expression of digital codes) causally operate within the restrictions imposed by physical and thermodynamic constraints. There is nothing mysterious about them. In the process of enzyme substrate recognitions new structures are formed and degraded that give rise to a network that is able to stabilize. However the production of polymers that help to specify the structures of enzymes facilitates the maintenance and as a by product their continuous production, making from then onwards the replication of templates a critical step that opened the way to Darwinian evolution.

Analog interactions between organisms and external factors are agent's responses to local and immediate challenging external circumstances in the form of structural adjustments. This form of analog information is the sufficient and necessary condition for selection of a digital record that registers the partial internalization of external referents by the agent. Digital records are created, replicated, mutated, selected, used and eliminated within the context of local interactions established with the surroundings. The random reorganization of previously encoded records provide the possibility to react successfully to unforseen situations. There will always be a number of undefined motifs that can be potentially recorded, and which one is to be incorporated in evolutionary time, into the digital record cannot be predicted beforehand. The origin of a digital description that has all the features of an abstract operation executed by physical systems results from their pragmatic drive to capture free energy and optimize its use conferring evolutionary advantage. Pragmatics is thus, the source of meanings and is always for the sake of someone. Zurek (1989, 1990) pointed out why the shorter the digital record the more efficient the extraction of energy by the agent and later Kauffman (2000) proposed to apply this idea to living systems as I have developed in my book (Andrade, 2003). The origin of semiosis lies in the fact that information processing is meaningful for the agents only as long as it is required for survival through energy capture and use.

Before I make the parallel between Collier's physical information system (PIS) and Zurek's Information Gathering and Using Systems (IGUS), I summarize Collier's

Hierarchical Dynamical Information Systems (HDIS) concept. Collier (2003) defines a HDIS as:

> A system of a number of relatively stable units that can combine more or less freely to form somewhat less stable structures has a capacity to carry information in a more or less arbitrary way. I call such a system a *physical information system* if its properties are dynamically specified. (Collier, 2003, page nu

He states that such systems are hierarchical with respect to the expression of lower level information at the higher level. While lower level information is syntactic, the expression at the higher level (dubbed biological information) is functional and has the capacity of development and evolution. Lower level information, he argues, is purely syntactical and can be treated without reference to meaning and is chemical (DNA, RNA, and proteins). I cannot agree with this viewpoint since RNA and proteins exhibit also three dimensional structures, a form that makes them liable of complementary interaction and chemically active as catalysts of chemical reactions. Even more of we consider that life may have originated from a preexisting RNA world or from a protein world then the existence of digital records in the form of DNA may be seen as an outcome of a previous evolutionary process and not a departing point. Higher level or biological information is embodied in the phenotype and also at an even higher level the species. There is also the emergence of new macro level information that is only potential in the lower level, but not specifically determined by it.

My tenet is that Zurek's IGUS are emergent systems because they fulfill the following Collier's requirements for PIS (Collier, 2003): 1) Non-reducibility to atomic components. As explained above this is a direct result of being open systems that are formed by internal structural interactions in the context of causal formative factors that are located in the environment. 2) Dynamical systems that can have effects on others. Information processing systems causally provoke effects on other systems, though they might not be deterministic. 3) Physically based. 4) They do work since they are far from equilibrium systems. In order to be able to act on something an expenditure of energy is required. 5) They are dissipative, a conclusion derived from the fact of being both open and far from equilibrium.

Thence, there is nothing mysterious about PIS. What I aim to point out is that Zurek's requirements for an IGUS are openness, memory, and being far from thermodynamic equilibrium. As mentioned above these three criteria are met in Collier's conditions. With openness to the environment comes unpredictability and novelty in emergence. Memory implies that the systems has an inner structure kept stable by cohesive interactions, though plastic enough that a plurality of microstates and macrostates can be attained, and finally the fact of being far form thermodynamic equilibrium ensures that it can do work and dissipate energy in the form of entropy.

Consequently IGUS also meet the conditions that define a nonholonomic non-Hamiltonian system. As dynamical emergent systems, living entities like IGUS display individuation and autonomy manifested in the process of establishing new interactions and the creation of informative records. They are thus semiotic agents. Autonomy, as Collier asserts means that such PIS use their own information to modify themselves and their environment to enhance their survival, responding to both environmental and internal stimuli to modify their functions to increase viability. The IGUS model not only accounts for what PIS accounts but goes beyond by stating that by means of agents' measurements and interactions, the establishment of structural adjustments provide the conditions for the creation and selection of digital records. In this manner, the path from dynamics to semiosis is really bridged.

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